UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4930 – Electricity Economics**

Date of exam: Friday, December 7, 2007 **Grades are given: January 4, 2008**

Time for exam: 9:00 a.m. – 12:00 noon

The problem set covers 3 pages

Resources allowed:

- No resources allowed

All questions should be answered, and Problem 1 and 2 count equally.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Problem 1

Consider the management of a pure hydropower system within a region (country) from the point of view of a social planner. Assume that the reservoir has an upper limit, and that the planning periods are $t = 1, \ldots, T$. Assume that variable costs are zero, and disregard any fixed costs. The social planning problem is:

$$\begin{aligned}
&\max \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)\,dz \\
&\text{subject to} \\
&R_t \le R_{t-1} + w_t - e_t^H \\
&R_t \le \bar{R} \\
&R_t, e_t^H \ge 0,\ t = 1, \ldots, T \\
&T, w_t, R_0, \bar{R} \text{ given}, R_T \text{ free}
\end{aligned}$$

where

$$\begin{aligned}
e_t^H &= \text{electricity production during period } t \text{ (kWh)} \\
p_t(e_t^H) &= \text{demand function for period } t \\
R_t &= \text{level of the reservoir at the end of period } t \text{ (kWh)} \\
w_t &= \text{inflow to the reservoir during period } t \text{ (kWh)} \\
\bar{R} &= \text{capacity of the reservoir (kWh)}
\end{aligned}$$

a) Give arguments for why discounting is disregarded in the optimisation problem. Derive the Kuhn-Tucker conditions and interpret the shadow prices. Make assumptions enabling qualitative analysis, and discuss the reasonableness of your assumptions.

b) Discuss the solution for the terminal period T using the assumptions you introduced above.

c) Discuss, using the Kuhn-Tucker conditions, how the reservoir constraint becoming binding in a period t can lead to a price change between period t and period t+1. Make assumptions about the price development from the future back to period t+2 such that water is transferred from period t+1 to period t+2. Show how the minimum level of the price from the future can be determined in order to give a non-negative transfer of water from period t+1 to period t+2. Try to illustrate this using a bathtub diagram as a window for period t and t+1 on the time axis, using linear demand curves for simplicity. Point out how the illustration conforms to the Kuhn-Tucker conditions for the periods involved.

d) Discuss how the reservoir becoming empty at the end of a period t can lead to a price change between period t and t+1. Illustrate using a bathtub diagram as a window for period t and t+1 on the time axis. Again, make assumptions about the price development from the future back to period t+1 such that water is transferred from period t+1 to period t+2. Point out how the illustration conforms to the Kuhn-Tucker conditions.

e) Discuss the following additional factors that can lead to changes in the social electricity price within the system over time for a planning horizon of two periods only (T = 2):
   i) A production constraint becomes binding in period 2
   ii) There is a transmission system enabling power to flow between producers and consumers (you may simplify to a single producer node and a single consumer node)
   iii) Uncertainty about inflow in period 2, but not in period 1.

   Each factor shall be discussed in isolation. The discussion should be brief, focussing on the economic explanations of price changes, and it is not the intention that you should use any mathematical model or set up bathtub diagrams answering these questions.

Problem 2

Consider that the hydropower system is run by a monopolist seeking to maximise the present value of profit. We do not specify discounting. The optimisation problem of the monopolist is:

$$\max \sum_{t=1}^{T} p_t(e_t^H)e_t^H$$

subject to

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$R_t, e_t^H \ge 0,\ t = 1,...,T$$

$$T, w_t, R_0, \overline{R} \text{ given, } R_T \text{ free}$$

a) Discuss the difference between the objective function of the social planner and the monopolist and derive the Kuhn-Tucker conditions for the monopolist. Specify reasonable assumptions for a qualitative analysis. Show the role played by the demand flexibility (the inverse of the demand elasticity) in characterising the difference between the social planning solution and the monopoly solution. Assuming that the monopoly solution is unique, what is the range for the price flexibility?

b) Discuss how the monopolist will use water, in the case of no spill and no binding reservoir constraint in any period, compared with the social solution for the same case. Derive an expression for the market price of the monopoly solution for each period, showing the role of the price flexibility.

c) Consider two periods only. Assume that it is optimal with spilling in period 1, but not in period 2. Explain why spilling in period 1, but not in period 2, must imply a binding reservoir constraint in period 1. Find the demand flexibility in the optimal solution for period 1, and show that the demand flexibility in period 2 is greater (absolute value less), evaluated at the optimal quantities. Discuss whether there is a conflict between this result and your conclusions in question b). Try to illustrate the case using a bathtub diagram. Assume that the reservoir constraint is binding in period 1 in the social planning case as well. Point out the correspondence between the Kuhn-Tucker conditions and the illustration.

d) Discuss the consequences of a regulator prohibiting spilling, building on an extension of the bathtub diagram you used answering question c). Comment upon the water value in period 1. Compare this regulated monopoly solution and the social solution.